Experimental scheme for unambiguous discrimination of linearly independent symmetric states

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We propose an optimal discrimination scheme for a case of four linearly independent nonorthogonal symmetric quantum states, based on linear optics only. The probability of discrimination is in agreement with the optimal probability for unambiguous discrimination among N symmetric states [Phys. Lett. A **250**, 223 (1998)]. The experimental setup can be extended for the case of discrimination among 2^M nonorthogonal symmetric quantum states.

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I. INTRODUCTION

A central problem in several quantum communication protocols, such as quantum cryptography [1, 2, 3], quantum teleportation [4] and entanglement concentration [5], is the discrimination among nonorthogonal quantum states, which can not be conclusively discriminated with von Neumann's measurements alone. For unambiguous discrimination among non-orthogonal quantum states it is necessary to use generalized quantum measurements. The discrimination process is error free in the case of a conclusive measurement, where the probability of obtaining an inconclusive result is non zero. In pioneer studies of Ivanovic-Dieks-Peres (IDP) [6], for finding the optimal probability of conclusive discrimination between two non-orthogonal quantum states, with equal *a priori* probability, they found that:

$$P_{IDP} = 1 - |\langle \Psi_+ | \Psi_- \rangle| \tag{1}$$

is the probability for error-free state discrimination, where $|\Psi_+\rangle$ and $|\Psi_-\rangle$ are the states being discriminated. Jaeger and Shimony have generalized this result by considering the case of states with different *a priori* probabilities [7]. Experiments for discriminating between nonorthogonal polarization states at IDP limit were accomplished utilizing linear optics only [8, 9]. In the same way, the experimental setup of unambiguous discrimination among three non-orthogonal quantum states was carried out by Mohseni *et. al* [10], with a success rate of 55%.

If we have a set of N non-orthogonal quantum states denoted as $\{|\Psi_k\rangle\}$, with $k = 0, ..., N-1$ lying in N-dimensional Hilbert space $\mathcal{H}$, there exists no a general strategy for unambiguous discrimination. If these states are linearly independent, it is possible to conclusively discriminate among them with a certain success probability. For this purpose, we need to extend the N-dimensional space at most to a dimension $2N-1$. This can be done by entangling the quantum system to a two-dimensional ancillary system (ancilla) [11]. After coupling the ancilla to the quantum system, usually under conditional evolution, a measurement over the ancilla projects the quantum system onto a state which depends on the result of ancilla's measurement. As we are dealing with a two dimensional ancilla, one of the results will allow conclusive discrimination of the original quantum state, and the other one gives an inconclusive measurement. If operators A_I and A_k describe the action on the quantum system in the cases of inconclusive and conclusive results, respectively, they must satisfy the relation

$$\sum_{k=0}^{N-1} A_k^\dagger A_k + A_I^\dagger A_I = \mathbf{1}. \tag{2}$$

In this article, we study the problem of discriminating non-orthogonal quantum states lying in a 2^M dimensional Hilbert space. For sake of simplicity, we describe an experimental setup in the case of dimension 4 ($M = 2$), which can be directly generalized to larger dimensional cases. The setup considers the generation process, propagation and discrimination of quantum states. We restrict ourselves to the case of non-orthogonal linearly independent states $\{|\Psi_k\rangle\}$ which are symmetric, defined by:

$$|\Psi_l\rangle = Z^l |\Psi_0\rangle, \tag{3}$$

where $|\Psi_0\rangle = \sum_{k=0}^{N-1} c_k |k\rangle$ is a normalized state, i.e., $\sum_{k=0}^{N-1} |c_k|^2 = 1$. The action of the Z operator on this state is such that $Z|k\rangle = exp(\frac{2\pi i k}{N})|k\rangle$ and $Z^N = I$. In ref. [12] the action of the conditional unitary evolution of a two-dimensional ancilla with the quantum system is written as :

$$U|\Psi_l\rangle \otimes |0\rangle_a = \sqrt{p_l}|u_l\rangle|0\rangle_a + \sqrt{1-p_l}|\phi_l\rangle|1\rangle_a.$$

where the $|0\rangle_a$ state is a known initial state of the ancillary system, the states $\{|u_l\rangle\}$ and $\{|\phi_l\rangle\}$ are orthogonal states and linearly dependent states, respectively, of the quantum system. In the case of measuring an ancilla in $|0\rangle_a$ state, the $|\Psi_l\rangle$ state is projected onto $|u_l\rangle$ state, with success probability p_l, which allows a conclusive discrimination with a von Neummann measurement in the basis

$\{|u_k\rangle\}$, since these states are orthogonal. In the case of the outcome $|1\rangle_a$ for the ancilla, the state of the system is projected onto linearly dependent states $\{|\phi_l\rangle\}$, which can not be unambiguously discriminated. In this process the optimal conclusive probability to discriminate between a set of N non-orthogonal symmetric states is $P_{opt} = N * min|c_k|^2$ [12], where c_k is the minimum coefficient, i.e., $|c_k| \leq |c_l|$ of state $|\Psi_0\rangle$ for $l = 0, 1, ..., N-1$.

This article has been organized as follows: In Sec. II we determine the conditional unitary transformation necessary for discrimination in the case of four non-orthogonal symmetric states. In Sec. III we describe an experimental setup for generating, propagating and discriminating among the four non-orthogonal states. This setup is based on down converted photons generated in a spontaneous down converted (SPDC) process . Finally, in Sec. IV we summarize our results and describe the application of them to several quantum communications protocols.

II. SYSTEM-ANCILLA CONDITIONAL EVOLUTION

Here, we consider the case of four non-orthogonal linearly independent symmetric states, which are denoted by $\{|\Psi_0\rangle, |\Psi_1\rangle, |\Psi_2\rangle, |\Psi_3\rangle\}$. These states are generated by applying the unitary transformation Z^l onto the $|\Psi_0\rangle$ state, such that $|\Psi_l\rangle = Z^l|\Psi_0\rangle$, with $l = 0, 1, 2, 3$. The $|\Psi_0\rangle$ state is defined by:

$$|\Psi_0\rangle = \sum_{k=0}^{3} c_k|k\rangle, \tag{4}$$

where the c_k coefficients obey the normalization condition and we will assume them to be reals. In general, these coefficients can be written as $c_0 = cos\theta_1$, $c_1 = cos\theta_2 sin\theta_1$, $c_2 = cos\theta_3 sin\theta_2 sin\theta_1$ and $c_3 = sin\theta_3 sin\theta_2 sin\theta_1$. The convenience of this notation becomes clear later on, when we discuss the physical implementation of the discrimination protocol. For building up the conditional unitary evolution, we will make use of the general approach proposed by He and Bergou [13], which allows to find a transformation that projects the $|\Psi_l\rangle$ states onto a set of orthogonal states $\{|u_l\rangle\}$ and onto another set of linearly dependent states, $\{|\phi_l\rangle\}$. Firstly, we must get the diagonal form of $A_I^\dagger A_I$ operators; this can be done when there exits a unitary operator U_o acting on the initial Hilbert space which gives:

$$U_o A_I^\dagger A_I U_o^\dagger = \sum_{i=0}^{D-1} \lambda_i |\alpha_i\rangle\langle\alpha_i|, \tag{5}$$

where $|\alpha_i\rangle$ is an eigenvector of the $A_I^\dagger A_I$ operator with eigenvalue λ_i. Since the $A_I^\dagger A_I$ operator is positive, its eigenvalues are defined between zero and one, and therefore we can define hermitian operators

$$A_I^\dagger = A_I = U_o^\dagger \sum_{i=0}^{D-1} \sqrt{\lambda_i}|\alpha_i\rangle\langle\alpha_i|U_o, \tag{6}$$

$$A_s^\dagger = A_s = U_o^\dagger \sum_{i=0}^{D-1} \sqrt{1-\lambda_i}|\alpha_i\rangle\langle\alpha_i|U_o. \tag{7}$$

The unitary transformation, in the enlarged space ancilla-system, takes the following form:

$$U = \begin{pmatrix} A_s & -A_I \\ A_I & A_s \end{pmatrix}. \tag{8}$$

where $A_s^\dagger A_s = \sum_{k=0}^{N-1} A_k^\dagger A_k$ is the operator corresponding a conclusive result. The U operator is not unique, there are three other similar forms [13]. We have assumed a qubit ancilla, with basis $\{|0\rangle_a, |1\rangle_a\}$ and initially prepared in the state $|0\rangle_a$. After the conditional evolution of the composite ancilla-system, the measurement on the ancilla giving the state $|0\rangle_a$ determines the action of the $A_k^\dagger A_k$ operator on the original quantum system, so that the discrimination process is conclusive. In the other case, the measurement on the ancilla is $|1\rangle_a$, the POVM element $A_I^\dagger A_I$ had acted on the quantum system and hence the discrimination process fails. An explicit form for the A_k operator was found by Chefles [5],

$$A_k = \frac{\sqrt{p_k}}{\langle\Psi_k^\perp|\Psi_k\rangle}|u_k\rangle\langle\Psi_k^\perp|, \tag{9}$$

where the $|u_k\rangle$ states form an orthonormal basis for $\mathcal{H}$; $|\Psi_k^\perp\rangle$ are the reciprocal states; and p_k is the probability to get the k-th outcome. This operator is consistent with

$$A_k|\psi_k\rangle = \sqrt{p_k}|u_k\rangle. \tag{10}$$

The reciprocal states $|\Psi_k^\perp\rangle$ are defined by

$$|\Psi_k^\perp\rangle = \frac{1}{\sqrt{q}} \sum_{r=0}^{N-1} \frac{1}{c_r^*} e^{\frac{2\pi i}{N}kr}|r\rangle, \tag{11}$$

where $q = \sum_j |c_j|^{-2}$ [12]. These states are also linearly independent and symmetric with respect to the Z transformation. Then operators A_s and A_I in case of discriminating $\{|\Psi_0\rangle, |\Psi_1\rangle, |\Psi_2\rangle, |\Psi_3\rangle\}$ states are:

$$\begin{aligned} A_s &= sin\theta_3 sin\theta_2 tg\theta_1|0\rangle\langle 0| + sin\theta_3 tg\theta_2|1\rangle\langle 1| \\ &+ tg\theta_3|2\rangle\langle 2| + |3\rangle\langle 3| \end{aligned} \tag{12}$$

and

$$\begin{aligned} A_I &= \sqrt{1 - sin^2\theta_3 sin^2\theta_2 tg^2\theta_1}|0\rangle\langle 0| \\ &+ \sqrt{1 - sin^2\theta_3 tg^2\theta_2}|1\rangle\langle 1| + \sqrt{1 - tg^2\theta_3}|2\rangle\langle 2|. \end{aligned} \tag{13}$$

Here, we have assumed that all *a priori* probabilities η_k to be equal, with a value $\frac{1}{N}$ and the discrimination probabilities to be $p_k = p_D$ [12].

After applying the conditional evolution on the compound ancilla-system, we get

$$U|\psi_l\rangle \otimes |0\rangle_a = \sqrt{p_D}|u_l\rangle|0\rangle_a + \sqrt{1-p_D}|\phi_l\rangle|1\rangle_a, \quad (14)$$

such that the symmetric states $\{|\Psi_0\rangle, |\Psi_1\rangle, |\Psi_2\rangle, |\Psi_3\rangle\}$ are projected to $\{|u_0\rangle, |u_1\rangle, |u_2\rangle, |u_3\rangle\}$ with a probability $p_D = 4 * |c_{\min}|^2$ when a projective measurement on the ancilla gives the $|0\rangle_a$ state, where $|c_{\min}| = \min\{|\cos\theta_1|, |\cos\theta_2\sin\theta_1|, |\cos\theta_3\sin\theta_2\sin\theta_1|,$ $|\sin\theta_3\sin\theta_2\sin\theta_1|\}$. For instance, in case of angles satisfying $0 \leq \theta_1 \leq \pi/3$, $0 \leq \theta_2 \leq 0.3\pi$ and $0 \leq \theta_3 \leq \pi/4$ the minimum coefficient is $|\sin\theta_3\sin\theta_2\sin\theta_1|$.

In this case, the orthogonal states $|u_l\rangle$ are found to be the four-dimensional Fourier transform acting on logical states $|l\rangle$, i.e, these states are given by:

$$|u_l\rangle = \mathcal{F} \mid l\rangle = \frac{1}{2}\sum_{k=0}^{3} e^{i\pi kl/2}|k\rangle, \quad (15)$$

Hence, the orthogonal states $|u_l\rangle$ are superpositions of the logical basis. We must apply the inverse of the Fourier transform for carrying out the discrimination among them in the logical basis which, in its matrix representation is given by:

$$\mathcal{F}^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}. \quad (16)$$

In terms of linear optics, this transformation can be regarded as a symmetric eight port beam splitter [14].

III. EXPERIMENTAL SETUP WITH TWO-PHOTON STATES

It is possible to implement the discrimination protocol by using single-photon states, where the logical states are defined by propagation paths. However, having a controlled source of single photons is rather difficult. Usually, for this purpose a highly attenuated pulsed laser is used, with a mean photon number less than one photon per pulse. For instance, ultralow intensity pulses are used for establishing quantum key distributions in cryptography experiments [15].

Here, we describe an experimental setup for implementing the optimal protocol for discriminating linearly independent quantum states, by using a simple optical system based on two-photon states generated in a spontaneous parametric down conversion process. The optimum is defined in the sense that the protocol maximizes the average success probability. We codify non-orthogonal quantum states in propagation paths in one of the down-converted photons (signal) and the other down-converted photon (idler) that will be used for coincidence measurement, i.e., this photon will ensure the presence of the other photon in one of the nonorthognal states. Thus, logical state $|j\rangle$, with $j = 0, 1, 2, 3$, corresponds to the j-th propagation path of the photon. The discrimination protocol is divided into four steps: preparation of the symmetric states; conditional ancilla-system evolution; projective measurement on the ancilla; and finally, in the case of conclusive measurement, discrimination of an orthogonal system's states.

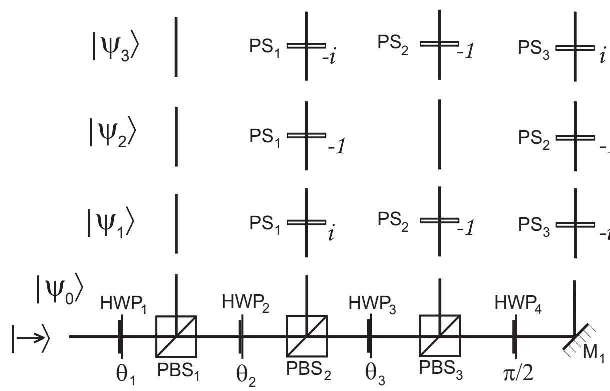

FIG. 1: Experimental setup for generating symmetric states in Eq. 4. In all the figures PBS, HWP and PS denote polarized beam splitter, half wave retardation plates, and phase shifter, respectively. The HWP$_j$ allows for rotating horizontal polarization in an angle θ_j, with these HWP and PBS the seminal state is generated, other states are simply generated by inserting PS in the propagation path of the photon.

As we have described above, generalized quantum measurements are implemented by embedding the quantum system into a large Hilbert space by adding an ancilla followed by an entangling operation. In this protocol, we use the polarization degree of freedom of the photon as our ancillary system. Hence, in the preparation stage of the symmetric states, we consider a photon initially prepared with horizontal polarization as input. Using half wave retardation plates (HWP), polarized beam splitter (PBS) and phase shifters (PS) we are able to generate the four symmetric states, see Fig. 1. The HWP$_i$ rotates the polarization of the photon in an angle $\pi/2 - \theta_i$. Hence, the vertical polarization of the photons is reflected at the PBS$_i$ and this component is used for defining the $|i-1\rangle$ logical state. The transmitted polarization goes trough the HWP$_{i+1}$. Actually, it is well known that, by using a HWP, a lossless PBS and a PS with appropriate parameters, any U(2) transformation can be implemented [17]. Considering that we have chosen the values of rotation angles at HWP's such that the minimum coefficient is $|\sin\theta_3\sin\theta_2\sin\theta_1|$, we generate the $|\Psi_0\rangle$ state. We remark that HWP$_4$ rotates the polarization of path 4 from horizontal to vertical polarization, so that at the end of the preparation stage the polarization of the photon is factorized from the path states, i.e., in all the propagation paths the polarization remains vertical. In the same way, other states $|\Psi_j\rangle$ are generated by inserting phase shifters, see Fig. 1.

The first step, in the discrimination protocol, is to

apply the conditional evolution (8) onto the symmetric states, which corresponds to a conditional rotation of the polarization (ancilla) depending on the propagation paths of the photon (logical states). Hence, the transformation is defined by its action on the logical states and the ancilla in $|0\rangle$ state:

$$\begin{aligned} U|00\rangle &= sin\theta_3 sin\theta_2 tg\theta_1|00\rangle + \sqrt{1-(sin\theta_3 sin\theta_2 tg\theta_1)^2}|01\rangle \\ U|10\rangle &= sin\theta_3 tg\theta_2|10\rangle + \sqrt{1-sin^2\theta_3 tg^2\theta_2}|11\rangle \\ U|20\rangle &= tg\theta_3|20\rangle + \sqrt{1-tg^2\theta_3}|21\rangle \\ U|30\rangle &= |30\rangle, \end{aligned} \tag{17}$$

which is implemented with HWP_5, HWP_6 and HWP_7. The optimum discrimination process is attained when we choose rotation angles at these HWP as $\theta_5 = \cos^{-1}(-\tan\theta_1\sin\theta_2\sin\theta_3)$, $\theta_6 = \cos^{-1}(-\tan\theta_2\sin\theta_3)$ and $\theta_7 = \cos^{-1}(-\tan\theta_3)$. The projective measurement is implemented right after applying the conditional evolution given by Eq. (17). Here, this is done by inserting polarized beam splitters PBS_4, PBS_5 and PBS_6 in propagation paths 0, 1 and 2, respectively, as is depicted in Fig. 2. An inconclusive measurement is obtained when a photon with horizontal polarization is transmitted through any one of these PBS. If this projective measurement gives a conclusive measurement (transmission of vertical polarization) that one of the $\{|u_l\rangle\}$ states has been transmitted, and for having a full discrimination, we need to determine which one of these $\{|u_l\rangle\}$ states has been transmitted.

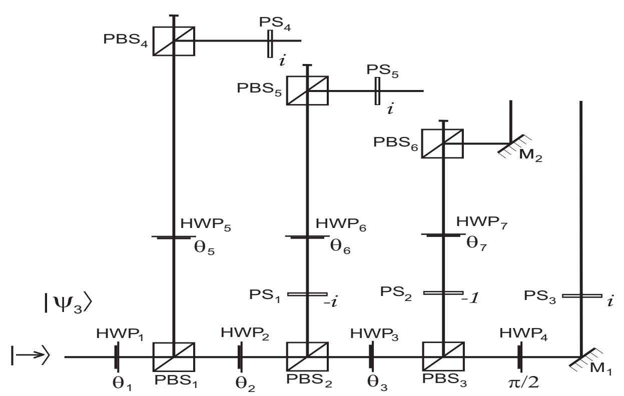

FIG. 2: Conditional evolution of the ancilla (polarization) depending on the logical states (propagation path) is achieved by inserting HWP in logical states $|0\rangle$, $|1\rangle$ and $|2\rangle$. The projection measurement on the ancilla is achieved by inserting PBS into the same propagation path, so that nonconclusive measurements are obtained when the photon is transmitted through one of these PBS.

Here, all the $\{|u_l\rangle\}$ states are orthogonal superpositions of the propagation paths, and each one of them is univocally associated with one of the non-orthogonal states. Hence, the last step in this protocol is the measurement of these orthogonal states, and for this purpose, it is convenient first to implement a unitary rotation satisfying $|l\rangle = \mathcal{F}^{-1}|u_l\rangle$, since in this case the discrimination is done by a detection of a photon propagating in path l. This unitary transformation is carried out using an eight port interferometer [14].

Number of states (2^M)	HWP	PBS	BS
4	7	6	4
8	15	14	12
16	31	30	32

TABLE I: Number of optical components for the discrimination protocol for different numbers of non-orthogonal states being discriminated. The total number of these components is approximately given by $2^M(M+2)$. The number of other optical components, such as mirrors and phase shifters, are of the same order.

The above described protocol is easily generalized to the case of 2^M symmetric states to be discriminated. In table I we listed the number of optical component as a function of the number of non-orthogonal states being discriminated.

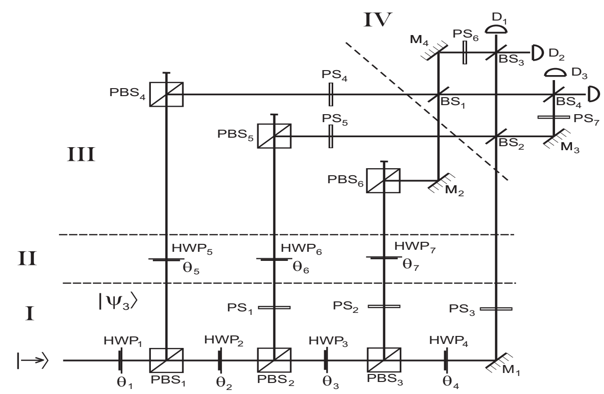

FIG. 3: General scheme for discrimination of the four symmetric states: (I) Preparation of state $||Psi_l\rangle$; (II) Conditional evolution of composite system; (III) Projection measurement; and (IV) detection. Here, the eight port interferometer has been inserted in the last stage of the experimental setup, which is in the right upper side of this figure.

We consider using use an Argon ion laser in a continuous wave operation, which pumps a β-Barium Borate nonlinear crystal with a power of 350 mW. The laser is made to operate in a single frequency mode at 351.1 nm, and the presence of other frequencies are eliminated by inserting a highly dispersive prism right after the laser. In addition, an interference filter of 10 nm, centered around 351.1 nm, is inserted into the propagation path of the pump field. Hence, two-photon states with center frequencies at 702.2 nm are generated. The nonlinear BBO crystal has been cut for SPDC type II, i.e, the propagation paths of down converted photons are non-collinear. We select signal (idler) photons linearly polarized in the horizontal (vertical) plane by inserting a Wallaston prism, with an extinction rate of $100,000:1$.

In the propagation path of the signal photon we insert the setup for implementing the discrimination protocol.

We assume that all the PBS have an extinction rate of 1,000:1. Controlled rotations of polarization states are accomplished by using HWP. In our case, the relative angle is adjusted to generate the appropriate coefficients c_k of symmetric states, Eq. (4). The purpose of the presence of PS appears to be evident after the projective measurement, due to the implementation of $\mathcal{F}^{-1}$ unitary transformations for mapping $|u_l\rangle$ states onto $|l\rangle$ states.

The eight port interferometer must be completely balanced and stabilized, where we deal with four interferometers in a Mach-Zehnder configuration. This can be done by a phase adjust mechanism on mirrors M_1 to M_4 . The angles and positions of these mirrors must be adjusted to optimize the interference fringes in the four output ports. For this purpose, mirrors and BS_1 to BS_4 must be mounted on precision translation stages, allowing the relative phase between the arms of each the Mach-Zehnder interferometer to be accurately varied. This stabilization process will be crucial for the discrimination protocol [18]. Here, we would like to remark that detectors D_1 to D_4 in the signal path are connected with detector D_i in the idler path for coincidence measurement.

IV. SUMMARY

We have proposed a scheme for the experimental discrimination of the four symmetric states. The protocol has been designed for obtaining the optimal value of conclusive measurements, which is given by the Chefless bound. Our scheme considers a reduced number of optical components and it can easily be generalized to the case of 2^N symmetric states. This, to the best of our knowledge, is the first proposal which can be generalized to larger dimensional quantum systems. The experimental setup is based on two-photon states from SPDC, which allows us to reach the optimal value for conclusive discrimination. Hence, by the transmission of a $|\Psi_l\rangle$ state at a time and coincidence measurement measuring between signal and idler photons, it is possible to obtain the conclusive probability p_D. The main experimental requirement is the stabilization of interferometers in Mach-Zehnder configurations.

We envisage the employment of the above described setup, for discriminating non-orthogonal symmetric states, for key distribution in a quantum cryptographic protocol. Recent works have demonstrated that cryptographic protocols are more robust against noise channels when using larger dimensional quantum systems [19]. For this purpose, the sender randomly chooses to generate one of the non-orthogonal states. In this case the propagation paths, after the generation stage, are coupled to single mode fiber optics, so that the polarization remains constant throughout the fiber. The receiver implements the discrimination protocol and the cases of conclusive measurement give a common element of the key to both the sender and the receiver. The presence of an eavesdropper, in between of authenticated users, can be detected in the authentication stage, where sender and receiver publicly announce a reduced number of the elements of the cryptographic key. Alternatively, this presence can also be noticed in a modification of the probabilities of nonconclusive measurement, which does not require a disclosing of part of the cryptographic key. This work is under study and we will publish elsewhere the study on the security of such a protocol. Besides, we also will study applying this protocol to the problem of discriminating between subsets of non-orthogonal quantum states, for this problem we will follow the work of Y. Sun *et. al* [20], where the case of a subset from three non-orthogonal states is studied.

V. ACKNOWLEDGMENT

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